

CE 272 – Traffic Network Equilibrium (3 credits)

Instructor: Tarun Rambha

Email: tarunrambha@iisc.ac.in

Timings and Location: TTh 8:30 – 10:00, Online

Course Website: <http://civil.iisc.ac.in/~tarunr/ce272.html>

Course Description:

This course will introduce the concept of equilibria in traffic networks (commonly referred to as the *traffic assignment problem*) which allows transportation planners to predict congestion or the volume of traffic on different roadway links and routes. These types of predictions are vital for short- and long-range transportation planning as they help make higher-level decisions related to the construction of new roads, selection of one-way streets, signal plan modifications, and toll collection. We will focus our attention on large-scale city networks as opposed to an intersection/junction or a corridor.

While the material in this course is specific to traffic networks, there are “themes” that are shared with a broad class of problems in which a large number of agents interact. To see these connections, we will make a few detours along the way and explore and borrow concepts that are native to game theory, graph theory, and mathematical optimization. Most of the topics covered in this course might also be relevant to electrical engineering and computer science students interested in *network games* and *algorithmic game theory*. Knowledge of elementary calculus and convex programming is helpful but no prerequisites are assumed. The course will be self-contained and the required background material will be covered in the class.

Reading Material:

The following books could be used as references. Additional reading material will be shared during the lectures.

1. Boyles, S. (2019). Transportation Network Analysis. [\[PDF\]](#)
2. Sheffi, Y. (1985). Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods. Prentice Hall. [\[PDF\]](#)
3. Patriksson, M. (2015). The traffic assignment problem: models and methods. Courier Dover Publications. [\[PDF\]](#)

Assignments:

The course will have both individual and group assignments. The written assignments will reinforce your understanding of the material presented in the class. All assignments must be typed and submitted electronically (of which the first two must be in \LaTeX for those in the research program). Students are encouraged to discuss the problems with their classmates but one must write their own solutions. **Plagiarism is strictly prohibited and will be penalized.**¹

In addition, the course will have a computer programming project with 3–4 major tasks. M.Tech. coursework students will also have to use a commercial software to run scenarios for a test network. The complete project is due on the day of the final. The objective of these tasks is to implement some of the algorithms introduced in this course and test it on real-world networks. You must submit your source code and some performance metrics using Python. The code must produce the correct output for any test instance and must include clear and concise comments on the methods and variables used.

Examinations:

¹<http://www.iisc.ac.in/about/student-corner/academic-integrity/>

The course will have one mid-semester and one comprehensive end-semester examination. Both examinations are in-class and cheat sheets will be allowed.

Grading:

Students will be graded on a 100-point scale. The weights for different components of the course areas follows: written assignments (30%), mid-semester exam (20%), and end-semester exam (50%, of which 30% is for the written section and 20% is for the project).

Lesson Schedule:

Table 1 lists the topics that will be covered in different lectures. Minor adjustments may be made as the semester progresses. Each week, lecture slides for the subsequent week will be uploaded on the website and you are encouraged to read the posted material before coming to the class. In Part I, we will review some background material, study the fundamentals of traffic equilibria, and train ourselves to look at equilibrium in multiple ways. In Part II, we will relax some of the underlying assumptions that were made in Part I to analyze variants that are somewhat more realistic. Finally, in Part III, we will explore advanced topics that include faster algorithms for computing the equilibrium solution. Due to time constraints, proofs of convergence of the algorithms used in this course will be skipped but are interesting topics for individual study.

Table 1: Course Schedule.

Module I - Background	
Lecture 1	Game Theory and Traffic Equilibria
Lecture 2	Review of Convex Optimization - Part I
Lecture 3	Review of Convex Optimization - Part II
Lecture 4	Fixed Point Theorems and Variational Inequalities
Lecture 5	Notation and Shortest Path Algorithms
Module II - Foundations	
Lecture 6	Wardrop User Equilibrium and Beckmann Formulation
Lecture 7	System Optimum, Price of Anarchy, and Congestion Pricing
Lecture 8	Method of Successive Averages
Lecture 9	Frank-Wolfe Algorithm
Lecture 10	Connections with Potential Games
Module III - Variants of the Traffic Assignment Problem (TAP)	
Lecture 11	Traffic Assignment with Elastic Demand - Part I
Lecture 12	Traffic Assignment with Elastic Demand - Part II
Lecture 13	Multi-Class User Equilibrium - Part I
Lecture 14	Multi-Class User Equilibrium - Part II
Module IV - Advanced Variants of the Traffic Assignment Problem (TAP)	
Lecture 15	Bi-Criterion Shortest Paths
Lecture 16	Bi-Criterion Traffic Assignment - Part I
Lecture 17	Bi-Criterion Traffic Assignment - Part II
Lecture 18	Overview of other TAP Variants
Module V - Faster Algorithms	
Lecture 19	Path Flows and Maximum Entropy
Lecture 20	Gradient Projection - Part I
Lecture 21	Gradient Projection - Part II
Lecture 22	Acyclic Graphs, Bushes, and Topological Ordering
Lecture 23	Algorithm B - Part I
Lecture 24	Algorithm B - Part II
Lecture 25	Sensitivity Analysis - Part I
Lecture 26	Sensitivity Analysis - Part II and Wrap-up