CE 272
Traffic Network Equilibrium

Lecture 24

Algorithm B - Part II

Previously on Traffic Network Equilibrium...

To topologically order nodes, define the indegree of a node as the number of arcs coming into it. Consider a node i with zero indegree. Set $\vartheta_i = 1$.

Delete the arcs emanating from i. Pick a new node with indegree zero and set it's topological order to 2.

Repeat this procedure until there are no nodes with zero indegree. If there are leftover nodes, then the network is acyclic.

Previously on Traffic Network Equilibrium...

In the 'Equilibrate Bush' step, we try to satisfy the Wardrop principle for a **given origin and all destinations** by routing flows along different paths **only using the Bush arcs**.

The flow shifts are similar to gradient projection where we move travelers from longest to shortest PASs using Newton's method.

Previously on Traffic Network Equilibrium...

In the equilibrate bush step, first

1. Compute link travel times
2. Compute link travel time derivatives
3. Find a topological ordering of the bush
4. Solve the shortest path problem
5. Solve the longest path problem

For the last two steps, find the distance and predecessor labels.

Previously on Traffic Network Equilibrium...

After finding the shortest and longest path labels and predecessors, the following steps are carried out.

1. Pick a node j with the highest topological order.
2. Use the shortest and longest paths to j to identify the last common node or divergence node i.
3. Determine the PASs using the shortest and longest sub-paths $\underline{P}_{ij}$ and $\overline{P}_{ij}$.
4. Calculate flow shift using Newton's method.
5. Update origin-based flows for the bush.
6. Set j to the previous node topologically and repeat from Step 2 until the origin r is reached.

Previously on Traffic Network Equilibrium...

After shifting flows in 'Equilibrate Bush', we get an ϵ-equilibrium bush. We can then update the link travel times for the new flows as seen earlier.

In the 'Optimize Bush' step, we try to update the bush by including shorter paths that are not a part of the current bush.

This step is carried out by

- Removing unused arcs
- Adding new shortcuts

Previously on Traffic Network Equilibrium...

The second option is to use the longest path labels instead of the shortest paths.

A non-bush link $(i, j) \notin B^r$ is added to the bush only if $\nu_j > \nu_i + t_{ij}$.

It is easy to show that this approach does not induce cycles even if the bush B^r isn't fully equilibrated.

Lecture Outline

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Algorithm B

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Initialization

In the previous class, a bush for the origin node was assumed to be known.

How can we obtain a bush and a feasible assignment to begin with?

1. Solve the one-to-all shortest path problem for an origin. The arcs belonging to the shortest paths will form a tree. (Why?)
2. Load all travelers from the origin along shortest paths.
3. Update travel times.
4. Repeat 1-3 for all origins.

Algorithm B

Complete Pseudocode

ALGORITHM B(G)

```
INITIALIZE BUSHES (G)
while Relative Gap > 10^-4 do
    for r ∈ Z do
        EQUILIBRATE BUSH (G, r)
        Update link travel times
    end for
    for r ∈ Z do
        Compute longest path labels using only bush links
        OPTIMIZE BUSH (G, r)
    end for
    Compute TSTT and SPTT
    Relative Gap ← TSTT/SPTT − 1
end while
```

Algorithm B

Other Bush-based Algorithms

Two other bush-based algorithms are popular:

- **Origin-based Assignment (OBA)**

  Bar-Gera, H. (2002). Origin-based algorithm for the traffic assignment problem. Transportation Science, 36(4), 398-417.

- **Linear User Cost Equilibrium (LUCE)**

  Gentile, G. (2009). Linear User Cost Equilibrium: a new algorithm for traffic assignment. Transportation Research B, 101.

Both these algorithms shift flows from non-basic PASs to a basic PAS within each equilibrate step unlike Algorithm B, which shifts flow only between longest and shortest paths.

Algorithm B is used in the commercial software TransCAD (Caliper) and LUCE is used in VISUM (PTV).

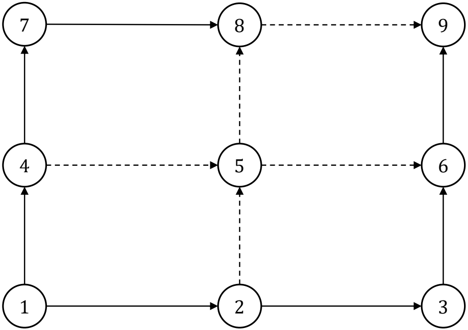
Lecture Outline

Example

Example

Network with 2 OD Pairs

Using the worksheets provided, solve for equilibrium in the following network using Algorithm B.



The delay functions on the thick links is $3 + (x/200)^2$, and that on the dashed links is $5 + (x/100)^2$. The demand between OD pairs (1,9) and (4,9) is 1000 vehicles.

Supplementary Reading

Dial, R. B. (2006). A path-based user-equilibrium traffic assignment algorithm that obviates path storage and enumeration. Transportation Research Part B: Methodological, 40(10), 917-936.

Your Moment of Zen

Boring, USA

Bland, Australia

Dull, Scotland