CE 269
Traffic Engineering

Lecture 5

Lane Changing Models

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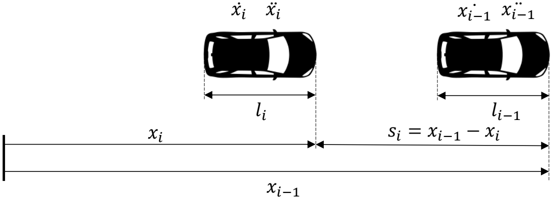
Throughout this lecture, we will refer to the lead vehicle using $i-1$ and the follower using i. The symbol x denotes the distance from a reference point and s_i indicates the spacing between the front ends of the vehicles.



The time gap of the following vehicle will be denoted using g_i and the distance between the rear end of the lead vehicle and front end of the following vehicle will be represented as g_i^x.

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Mathematically, let the acceleration of vehicle i, $\ddot{x}_i(s_i, v_i, \Delta v_i)$ be written as a function of the spacing s_i, velocity of the current vehicle v_i, and the speed differential $\Delta v_i = v_i - v_{i-1}$.

Dependence on t is not shown in the above expressions but is implicitly assumed. The required properties can be expressed as

- As vehicles travel faster, they tend to accelerate less

$$\frac{\partial \ddot{x}_i(s_i, v_i, \Delta v_i)}{\partial v_i} < 0$$

- If there is no vehicle in front, drivers prefer to travel at a desired speed

$$\lim_{s_i \to \infty} \ddot{x}_i(s_i, v_i^{max}, \Delta v_i) = 0$$

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- If a lead vehicle is far away, the following vehicle must accelerate

$$\frac{\partial \ddot{x}_i(s_i, v_i, \Delta v_i)}{\partial s_i} \geq 0, \lim_{s_i \to \infty} \frac{\partial \ddot{x}_i(s_i, v_i, \Delta v_i)}{\partial s_i} = 0$$

- Acceleration decreases with increase in speed differential

$$\frac{\partial \ddot{x}_i(s_i, v_i, \Delta v_i)}{\partial \Delta v_i} \leq 0$$

Lecture Outline

1. Discrete-Choice Based Models
2. MOBIL

Discrete-Choice Based Models

Discrete-Choice Based Models

So far, we have seen how to model the speeds and accelerations of vehicles using coupled ODEs that include variables from a lead vehicles.

Lane changing is another important action taken by vehicles in uninterrupted traffic. While lane changing can help individual drivers, it can result in congestion for upstream traffic.

Unlike the car-following models, lane changing involves discrete actions of whether to shift lanes or not. These maneuvers are complicated because they involve behavioural perceptions and responses to brake lights and indicators.

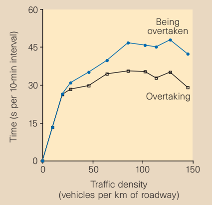
Discrete-Choice Based Models

The Grass is Green on the Other Side

Drivers often feel that the next lane is moving faster even if they have the same average speeds.

This can be tested using simple car following models without lane changes and by tracking the time spent overtaken and comparing it with the time spent overtaking.

This plot by Redelmeir and Tibshirani show these metrics for every 10 minutes of travel.



They also showed a video of slower moving traffic in an adjacent lane but about 70% of the participants felt that the next lane was moving faster.

A potential reason is that vehicles which are overtaken are visible to the driver only for a short time compared to those that overtake the driver.

Discrete-Choice Based Models

Introduction

Lane change models are generally classified as

- Mandatory Lane Changing
- Discretionary Lane Changing

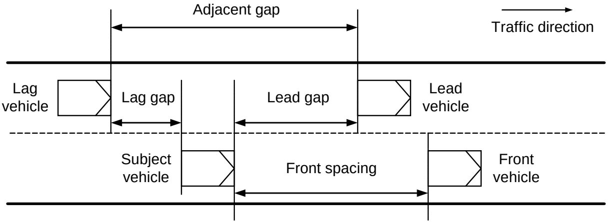
Mandatory lane changes occur when a driver has to exit from a highway facility, or avoid a work zone, or merge onto a highway.

Discretionary lane changes on the other hand are done to avoid trailing behind slow moving vehicles or heavy vehicles.

Discrete-Choice Based Models

Target Lane Model

What are the factors that influence decisions of drivers to change lanes?



- Lead and lag gaps
- Speed differences between the subject and other vehicles
- Type of subject and lead/lag vehicles
- Keep left/right rules

Imagine a driver i can choose from three actions $TL = CL, RL, LL$ that indicate the current lane, right lane, and left lanes. Time is typically discretized into a finite number of intervals.

The utility of a driver for choosing a particular lane can be written as

$$U_i^{TL}(t) = \boldsymbol{\beta}^{TL} X_i^{TL}(t) + \alpha^{TL}\nu_i + \epsilon_i^{TL}(t)$$

where $X_i^{TL}(t)$ is a vector of explanatory variables that capture the neighbourhood of a driver and additional variables associated with their path such as highway exits.

ν_i is constant across time but is different for different times. It captures correlations between observations and is assumed to be Gaussian across the population with parameter α^{TL}.

Discrete-Choice Based Models

Target Lane Model

Assuming that ϵs at each time step for each driver are iid Gumbel distributed, the conditional probability of choosing a lane given the individual-specific error term is

$$\mathbb{P}_i(TL|\nu_i) = \frac{\exp(\boldsymbol{\beta}^{TL} X_i^{TL}(t) + \alpha^{TL}\nu_i)}{\sum_{TL'} \exp(\boldsymbol{\beta}^{TL'} X_i^{TL'}(t) + \alpha^{TL'}\nu_i)}$$

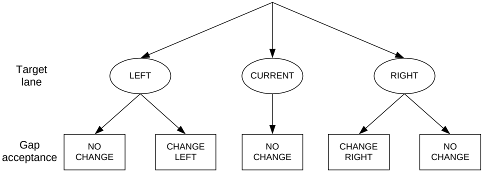
One of the α values has to be normalized to zero to avoid identification issues.

Once the driver decides to change to a particular lane, they must also check if the gap is sufficient for a lane change.

Discrete-Choice Based Models

Gap Acceptance Model

Different drivers may be comfortable at changing lanes at different thresholds. These are captured using distance gaps.



Critical gaps can be estimated empirically. Since gaps are positive quantities, the critical values are assumed to follow a log-normal distribution.

$$\ln(g_i^{x,cr,lead,TL}(t)) = \beta^{lead} X_i^{lead,TL}(t) + \alpha^{lead}\nu_i + \epsilon_i^{lead}(t)$$

where TL is the target lane which can either be LL or RL and ϵs are Gaussian. A similar expression can be written for the critical lag gap.

Discrete-Choice Based Models

Gap Acceptance Model

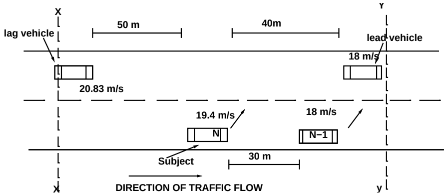
Suppose $I^{TL}(t)$ represents a random variable which takes 1 if a lane change to TL is executed at time t.

The probability that a driver i will change to a target lane TL given ν_i, $\mathbb{P}(I^{TL}(t) = 1|\,TL, \nu_i)$, therefore equal to

$$
\begin{aligned}
\mathbb{P}\left[g_i^{x,lead,TL}(t) > g_i^{x,cr,lead,TL}(t)|\,TL, \nu_i\right] \mathbb{P}\left[g_i^{x,lag,TL}(t) > g_i^{x,cr,lag,TL}(t)|\,TL, \nu_i\right] \\
= \Phi\left(\frac{\ln(g_i^{x,lead,TL}(t)) - \beta^{lead} X_i^{lead,TL}(t) - \alpha^{lead}\nu_i}{\sigma^{lead}}\right) \\
\Phi\left(\frac{\ln(g_i^{x,lag,TL}(t)) - \beta^{lag} X_i^{lag,TL}(t) - \alpha^{lag}\nu_i}{\sigma^{lag}}\right)
\end{aligned}
$$

The subscript i is being ignored in some of the terms since it is clear from the context.

For the scenario shown in the following figure, find the probability of changing lanes



Assume that

- $\sigma^{lead} = \sigma^{lag} = 2$
- $\beta^{lead} = \beta^{lag} = 1$
- $X_i^{lead,TL}(t) = X_i^{lag,TL}(t) = 0.8$
- $\nu_i = 0.7$ and $\alpha^{lead} = \alpha^{lag} = 1.2$

Discrete-Choice Based Models

Joint Model

Note that from data, we can only observe the final lane changing action. Mathematically, we can estimate the following marginal probability,

$$\mathbb{P}_i(l^{TL}(t)|\nu_i) = \sum_{TL} \mathbb{P}_i(TL, l^{TL}(t)|\nu_i) = \sum_{TL} \mathbb{P}_i(TL|\nu_i)\mathbb{P}_i(l^{TL}(t)|TL, \nu_i)$$

How to construct a likelihood function? The decisions at different time steps are treated as independent events and hence we find

$$\prod_t \mathbb{P}_i(l^{TL}(t)|\nu_i)$$

The unconditional likelihood function can then be written as

$$\mathcal{L}_i = \int_\nu \prod_t \mathbb{P}_i(l^{TL}(t)|\nu_i)d\Phi(\nu_i)$$

This can be summed across drivers to estimate parameters.

Discrete-Choice Based Models

Mandatory Lane Changes

Mandatory lane changes can be modeled differently since the action must be complete at a certain distance.

For instance Yang and Koutsopoulos suggest that the probability of changing lanes at a distance x_i away is given by

$$p_i = \begin{cases} \exp\left((x_i - x_0)^2/\sigma_n^2\right) & x_i > x_0 \\ 1 & x_i \leq x_0 \end{cases}$$

where x_0 is the critical distance beyond which a lane change is inevitable.

The parameter $\sigma_i = \alpha_0 + \alpha_1 m_i + \alpha K$ is calibrated from data where m_i is the number of lanes between the current lane and the target lane and k is traffic density. The acceptable gaps are given by another rule-based formula.

Lecture Outline

MOBIL

MOBIL

Introduction

Minimizing overall braking deceleration induced by lane changes (MOBIL) is another popular lane changing model which is relatively simpler.

This model also selects the action that results in maximum utility, but the utility is deterministic and is expressed in terms of accelerations.

Additionally, the model makes safety checks to prevent collisions and unreasonable decelerations of the subject and other vehicles

A major advantage of the MOBIL model is that it can easily be plugged into other car-following models.

MOBIL

Safety Criterion

Suppose i represents the subject vehicle and let j denote other vehicles (lead/lag vehicles) in the adjacent lane.

A lane change is carried out only if $a_j > -b$ for all vehicles where b is the comfortable deceleration rate, similar to the one used in IDM.

The acceleration of the lag vehicle in the adjacent vehicle should be updated assuming that the subject vehicle made a lane change before checking for this criterion.

MOBIL

Incentive Criterion

The target lane which allows the vehicle to accelerate the most is chosen.

$$TL^* = \arg\max U_i(TL) = \arg\max a_i^{TL}$$

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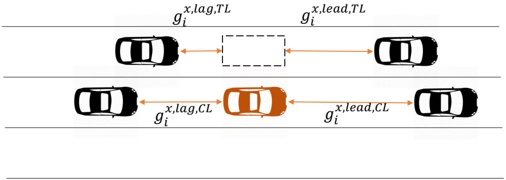

However, this criterion can induce frequent lane changes. To avoid such artifacts, it is modified as

$$a_i^{TL} - a_i^{CL} + p\ (\text{Difference in accelerations of lag vehicles}) > \Delta a + a_{bias}$$

MOBIL

Incentive Criterion

Parameter	Typical value
Limit for safe deceleration b_{safe}	$2\,\text{m/s}^2$
Changing threshold Δa	$0.1\,\text{m/s}^2$
Asymmetry term (keep-right directive) a_{bias}	$0.3\,\text{m/s}^2$
Politeness factor p (MOBIL lane-changing model)	0.0–1.0

For the lane changing example shown earlier, use the GM car following model and check if the safety criterion and incentive criterion are satisfied.

Your Moment of Zen