CE 205A
Transportation Logistics

Lecture 15

Bounds and Heuristics for TSP

Previously on Transportation Logistics

The DFJ formulation using E-δ notation can be written as

$$\begin{aligned} \min \quad & \sum_{e \in E} c_e x_e \\ \text{s.t.} \quad & x(\delta(u)) = 2 && \forall\, u \in V \\ & x(E(S)) \leq |S| - 1 && \forall\, S \subset V, S \neq \emptyset \\ & x_e \in \{0, 1\} && \forall\, e \in E \end{aligned}$$

The formulation with the alternate SEC constraints take the form

$$\begin{aligned} \min \quad & \sum_{e \in E} c_e x_e \\ \text{s.t.} \quad & x(\delta(u)) = 2 && \forall\, u \in V \\ & x(\delta(S)) \geq 2 && \forall\, S \subset V, S \neq \emptyset \\ & x_e \in \{0, 1\} && \forall\, e \in E \end{aligned}$$

We can further restrict $3 \leq |S| \leq |V|/2$. (Why?)

Lecture Outline

Bounds for TSPs

Bounds for TSPs

Introduction

Upper bounds

- LKH Heuristic
- Christofides-Serdyukov algorithm
- Nearest neighbour

Lower bounds

- Euclidean instances
- Lagrangian relaxation

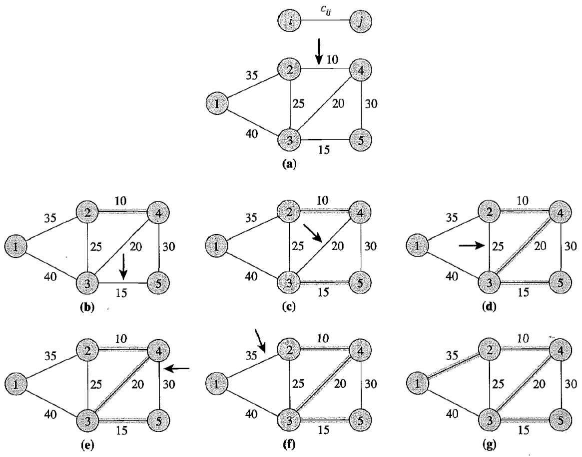
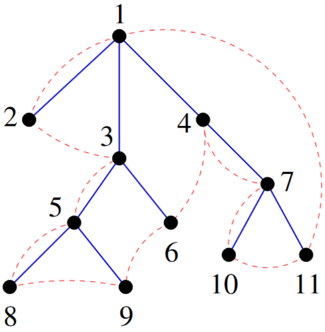


Figure 13.6 Illustrating Kruskal's algorithm.



Show that

$$MST \leq TSP \leq 2MST$$

This is hence called a 2-approximation algorithm. Christofides exploited this technique to develop a 3/2-approximate algorithm.

Lecture Outline

Heuristics

Heuristics

2-opt

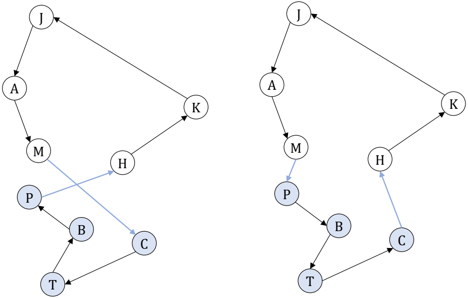
Exact approaches involve adding cuts the LP relaxation and using a Branch-and-Cut scheme.

Alternately, one could design heuristics that generate other tours from a given tour and checks if the new solution has a lesser cost. Some commonly used methods are:

- Flip Routines
- Insertion Routines

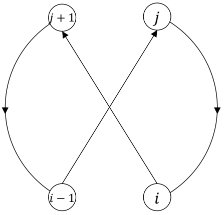
Suppose we flip the set of cities shown in blue in the following tour.

Jaipur–Ahmedabad–Mumbai–Chennai–Trivandrum–Bangalore–Panaji–Hyderabac
Kolkata–Jaipur

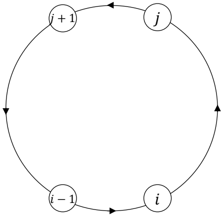


Jaipur–Ahmedabad–Mumbai–Panaji–Bangalore–Trivandrum–Chennai–Hyderabac
Kolkata–Jaipur

Flipping routines can be viewed as replacing two edges of the original tour with two new edges.



$\ldots, i-1, j, \ldots, i, j+1, \ldots$



$\ldots, i-1, i, \ldots, j, j+1, \ldots$

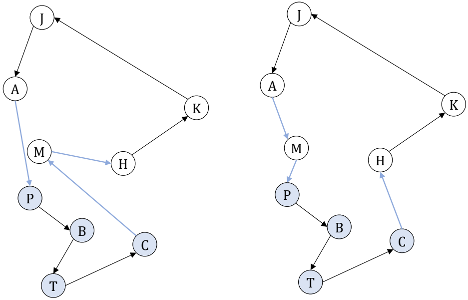
Consider the symmetric TSP. The routine `flip`(j, i) is profitable only if

$$c_{i-1,j} + c_{i,j+1} \geq c_{i-1,i} + c_{j,j+1}$$

A solution is said to be *2-opt* if we cannot get a cheaper tour by replacing 2 edges in the current tour with another set of 2 edges.

Alternately, we can insert tour segments at different locations. For example, consider the tour

Jaipur–Ahmedabad–Panaji–Bangalore–Trivandrum–Chennai–Mumbai–Hyderabad
Kolkata–Jaipur

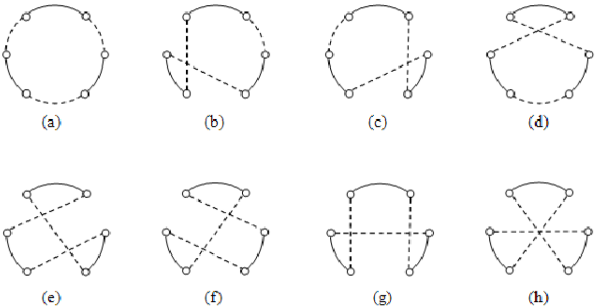


Remove the blue segment and insert it between Mumbai and Hyderabad

Heuristics

3-opt

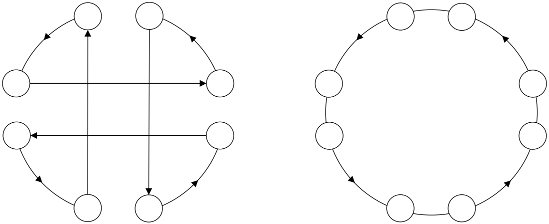
Notice that in the process of inserting tour segments, we had to replace 3 edges in the original tour with 3 new edges.



Our earlier analysis can be extended to 3-opt solutions which cannot give cheaper tours by replacing 3 edges with another set of 3 edges.

Some of the 3-opt moves can be obtained by sequential 2-opt moves, e.g., (b)-(g) (although we don't know if the 2-opt moves are profitable).

The analysis can be extended to *k-opt* moves but the benefits tend to decrease with increase in k.



Lin-Kernighan heuristic is one of the fastest TSP heuristic which uses variable k-opt moves ([**?**])

Heuristics

Simulated Annealing

Annealing involves heating a material to alter its physical properties and then cooling it back.

At higher temperatures, the molecules in the material move freely and at lower temperature they are restrained.

Similar ideas have been used to design heuristics where a wide search is performed in the beginning and sub-optimality is encouraged.

However, as iterations pass, the temperature is reduced gradually and non-optimal solutions are discarded.

Heuristics

Simulated Annealing

Suppose x' is a neighbour of solution x and let $f(.)$ denote the objective function.

If $f(x') \leq f(x)$, update $x \leftarrow x'$. Otherwise, update $x \leftarrow x'$ with a probability equal to $\exp(-(f(x') - f(x))/T)$.

The temperature values are reduced gradually according to a cooling schedule.

Fig. 9. Results at four temperatures for a clustered 400-city traveling salesman problem. The points are uniformly distributed in nine regions. (a) $T = 1.2$, $\alpha = 2.0567$; (b) $T = 0.8$, $\alpha = 1.515$; (c) $T = 0.4$, $\alpha = 1.055$; (d) $T = 0.0$, $\alpha = 0.7839$.

Genetic algorithms are bio-inspired and maintain a population of solutions at each stage.

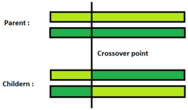
The solutions are then bred to create child solutions or offsprings from parent chromosomes and their objective or fitness functions are evaluated.

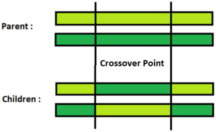
With a small probability, the population is also mutated to induce random changes which facilitates the exploration of the solution space.

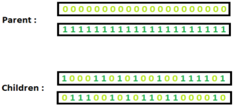
Heuristics

Genetic Algorithms

New solutions are created using 1-point, 2-point crossovers, and uniform crossovers.



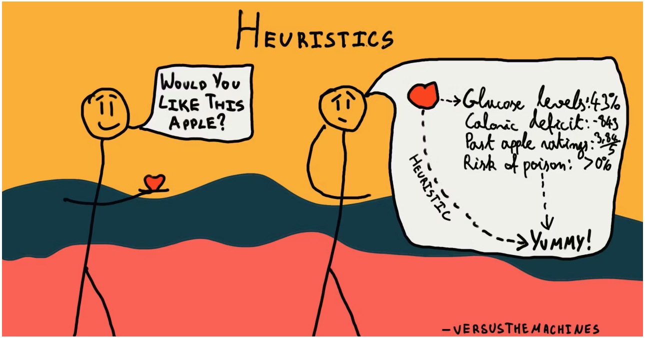




Different problems requires different ways of representing chromosomes. How would you represent the solution of a facility location problem?

Image Source: GeeksforGeeks

Your Moment of Zen



Source: thedecisionlab